AP Calculus AB | Homework 0210 | Name:
Dr. Paul L. Bailey | Monday, February 10, 2025

This is the first of our "spiraling review" homework sets. We will have daily homework assignments consisting of ten problems. Some of the problems will practice new material, and the others will be review. The new section in Thomas will be indicated on the homework; please *read the book*.

Write your homework *neatly*, *in pencil*, on $8\frac{1}{2} \times 11$ blank white printer paper (the back can be used). Always *write the problem*, or at least enough of it so that your work is readable. In particular, you *must* write any function the problem refers to.

Learn to *write in sentences*. Use words, sentences, paragraphs when appropriate. Sentences begin with a word and end with a period. Avoid having apparently random expressions and equations scattered around the page. Justify your conclusions.

You should memorize all relevant definitions and understand all relevant theorems, as soon as possible after we first cover them. The current new theory is stated below.

Definition 1. Let $D \subset \mathbb{R}$ and let $f : D \to \mathbb{R}$. Let $c \in D$.
We say that f has a *global maximum* at c if $f(c) \geq f(x)$ for all $x \in D$.
We say that f has a *global minimum* at c if $f(c) \leq f(x)$ for all $x \in D$.
We say that f has a *global extremum* at c if f has a global minimum or maximum at c.
A synonym for global in this context is *absolute*.

Theorem 1. Extreme Value Theorem (EVT)
Let $f : [a, b] \to \mathbb{R}$ be continuous. Then there exist $c_1, c_2 \in [a, b]$ such that f has a global minimum at c_1 and a global maximum at c_2.

Definition 2. Let $D \subset \mathbb{R}$ and let $c \in D$.
We say that c is an *interior point* of D if there exists $\delta > 0$ such that $(c - \delta, c + \delta) \subset D$.
A *neighborhood* of c is a set which contains an open interval which contains c.

Definition 3. Let $D \subset \mathbb{R}$ and let $f : D \to \mathbb{R}$. Let $c \in D$ be an interior point of D.
We say that f has a *local maximum* at c if $f(c) \geq f(x)$ for all x in some open interval containing c.
We say that f has a *local minimum* at c if $f(c) \leq f(x)$ for all x in some open interval containing c.
A synonym for local in this context is *relative*.

Theorem 2. *If f has a local extremum at c, and $f'(c)$ exists, then $f'(c) = 0$.*

Definition 4. Let $D \subset \mathbb{R}$ and let $f : D \to \mathbb{R}$. Let $c \in D$ be an interior point of D.
We say that c is a *critical point* of f if either $f'(c) = 0$, or if $f'(c)$ does not exist.

Problem 1 (Thomas §4.1 # 18). Let $f : [-3, 1] \to \mathbb{R}$ be given by $f(x) = 4 - x^2$. Find the absolute maximum and minimum values of f. Graph the function, identifying the points of the graph where the absolute extrema occur, and include their coordinates.

Problem 2 (Thomas §4.1 # 23). Let $f : [-2, 1] \to \mathbb{R}$ be given by $f(x) = \sqrt{4 - x^2}$. Find the absolute maximum and minimum values of f. Graph the function, identifying the points of the graph where the absolute extrema occur, and include their coordinates.

Problem 3 (Thomas §4.1 # 43). Let $f(x) = \dfrac{x}{x^2 + 1}$. Find the extreme values of f and where they occur. Find the domain and range of f.

Problem 4 (Thomas §4.1 # 48). Let $f(x) = x^2\sqrt{3 - x}$. Find all critical points of f. Determine the local extreme values of f.

Problem 5 (Thomas §2.6 # 35). Define $g(3)$ in a way that extends $g(x) = (x^2 - 9)/(x - 3)$ to be continuous at $x = 3$.

Problem 6 (Thomas §3.8 # 9). If f is differentiable at a, the *linearization* of f at a is the function

$$L(x) = f'(a)(x - a) + f(a).$$

Find the linearization of $f(x) = \sqrt[3]{x}$ at $a = 1$, and use it to approximate $f(1.3)$.

Problem 7 (APCalcAB.1969.MC.8). Let $p(x) = (x + 2)(x + k)$. Suppose that the remainder is 12 when $p(x)$ is divided by $x - 1$. Find k. (Hint: use the Remainder Theorem.)

Problem 8 (APCalcAB.1969.MC.18). Let $f(x) = 2 + |x - 3|$. Find $f'(x)$ and $f'(3)$.

Problem 9. Compute

$$\frac{d^{999}}{dx^{999}} \sin x.$$

Problem 10. Let H denote the set of all henhouses in the United States, and let C denote the set of all chickens that live in one henhouse. Let

$$f : C \to H \quad \text{be given by} \quad f(\texttt{chicken}) = \texttt{henhouse in which chicken lives}.$$

Let

$$g : H \to \mathbb{R} \quad \text{be given by} \quad g(\texttt{henhouse}) = \texttt{area in square footage of the henhouse}.$$

Let

$$h : C \to \mathbb{R} \quad \text{be given by} \quad h(c) = \frac{g \circ f(c)}{|f^{-1}(f(c))|}.$$

Suppose $c \in C$. Describe $h(c)$.